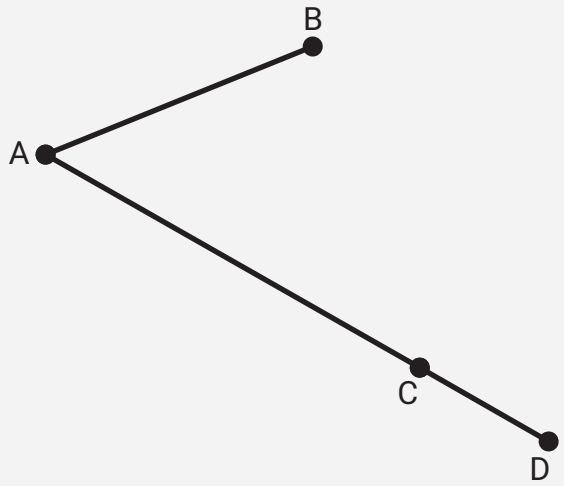
Mathematics Level 4

Measurement and Geometry

Example: Geometry

The coordinates of two points A and B are respectively (2, 3) and (7, 5).



The points A, C and D lie on the same line.

The line intervals AC, AD and CD are related by the equation AC × AD = 2 × CD^2.

The interval AC is 5 units long.

The diagram is **not** drawn to scale.

What is the length of the line segment AB?

A 7

B $\sqrt{7}$

C $\sqrt{29}$

D $\sqrt{145}$

ANSWER C: $\sqrt{29}\left(\sqrt{(5-3)^2+(7-2)^2}=\sqrt{9}\right)$

Students select option A: as they may simply add the vertical and horizontal lengths between the two points: vertical (5 — 3) and horizontal (7 — 2).

Students select option B: as they may not square the vertical and horizontal lengths $\left(\sqrt{(5-3)+(7-2)}=\sqrt{7}\right)$.

Students select option D: as they may add the coordinates instead of subtracting to determine vertical and horizontal lengths $\left(\sqrt{(5+3)^2+(7+2)^2}=\sqrt{64+81}=\sqrt{145}\right)$.

COMMENTARY

Students are required to determine the length of a line interval using the coordinates of the end points. Students will need to use the coordinates of the two points in the distance formula $\left(d=\sqrt{(y_1-y_2)^2+(x_2-x_1)^2}\right)$.

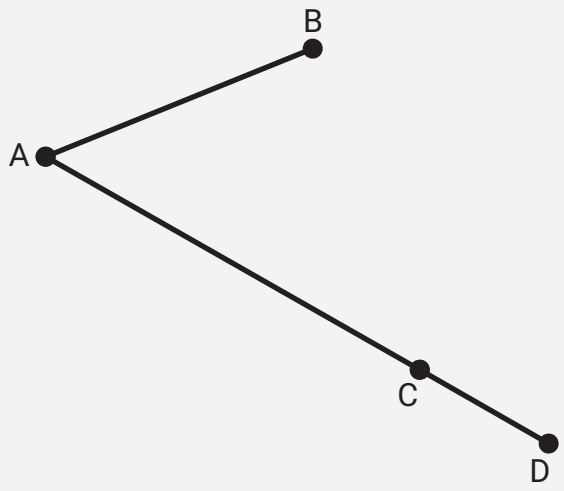
The distance formula is derived from Pythagoras's Theorem and students who are not familiar with the formula could instead use Pythagoras directly.

Mathematics Level 4

Measurement and Geometry

Example: Geometry

The coordinates of two points A and B are respectively (2, 3) and (7, 5).



The points A, C and D lie on the same line.

The line intervals AC, AD and CD are related by the equation AC × AD = 2 × CD^2.

The interval AC is 5 units long.

The diagram is **not** drawn to scale.

What is the length of the line segment AB?

A 1.25 units

B 2.5 units

C 5 units

D 10 units

ANSWER C: 5 units

(TD note: students could guess 5 because it's the same length as that given for AC. Having CD visibly look quite different could help to avoid that.)

Students select option A: as they may use the diagram and estimate the length of CD as 1/4 the length of AC.

Students select option B: as they may correctly setup and solve the quadratic equation but use the incorrect solution 2.5.

Students select option D: as they may correctly setup and solve the quadratic equation but assume that the solution 10 is the final answer.

COMMENTARY

Students are required to find an unknown length. The information given includes enough to allow the setup of a quadratic equation. Students will need to solve the equation, discard an invalid solution, and then use the valid solution to determine the final answer.

Students are given:

$$AC \times AD = 2 \times CD^2$$
$$AC = 5$$

Students need to use the fact that:

$$AD = AC + CD$$

which rearranges to

$$CD = AD - AC$$

And then:

$$AC \times AD = 2 \times (AD - AC)^2$$
$$5 \times AD = 2 \times (AD - 5)^2$$
$$5AD = 2 \times (AD^2 - 10AD + 25)$$
$$5AD = 2AD^2 - 20AD + 50$$
$$0 = 2AD^2 - 20AD + 50$$

Which can be solved using the Null Factor Theorem to give:

$$2AD^2 - 25AD + 50 = 0$$
$$(2AD - 5)(AD - 10) = 0$$
$$AD = 2.5 \text{ or } 10$$

Since AC = 5, the value 2.5 is meaningless and so the only valid solution is 10.

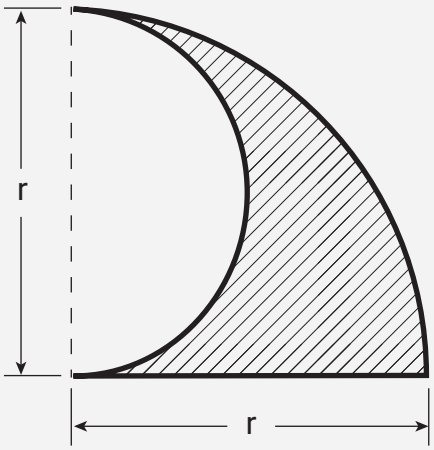
Finally this gives an answer for $CD = AD - AC = 10 - 5 = 5$ units

Mathematics Level 4

Measurement and Geometry

Example: Measurement

A surfboard fin is shaped by removing a semi-circle of diameter r from a quarter circle of radius r.



What is the shaded area shown – the area of one side of the fin?

A $\frac{1}{8}\pi r^2$

B $\frac{1}{4}\pi r^2$

C $\frac{1}{2}\pi r^2$

D πr^2

ANSWER A: $\frac{1}{8}\pi r^2$

Students select option B: as they may determine the area of the quarter circle.

Students select option C: as they may use r as the radius of the small semicircle and $2r$ as the radius of the quarter circle.

Students select option D: as they may apply the formula for area of a circle.

COMMENTARY

Students are required to find the area of an irregular shape by identifying it is part of a circle. This allows students to apply circle knowledge to the problem to find the area. Students need to note that 'r' is used to represent the radius of the full circle and this will need to be halved for the radius of the semicircle.

The pronumeral r describes two different characteristics in the composite shape – the diameter of one shape and the radius of another. Students will need to determine expressions for the areas of the quarter and semi circles and then combine them by subtraction.

Mathematics Level 4

Number and Algebra

Example: Number

$x = 1$ and $x = 2$ are two solutions of the equation $2^{2x} - a(2^x) + b = 0$

What are the values of a and b respectively?

A $\frac{10}{3}, \frac{-2}{3}$

B $\frac{10}{3}, \frac{-8}{3}$

C $1, 2$

D $6, 8$

ANSWER D: $6, 8$

Students select option A: as they may make a mistake in solving the simultaneous equations, substituting $a = \frac{10}{3}$ into $16 - 4a + b = 0$.

Students select option B: as they may make a mistake in solving the simultaneous equations, substituting $a = \frac{10}{3}$ into $4 - 2a + b = 0$.

Students select option C: as they may assume that the given solutions for x are also values for a and b.

COMMENTARY

Students are required to determine the values of two unknowns in an algebraic equation. Students could solve this by substituting the given x values into the equation $2^{2x} - a(2^x) + b = 0$ generating two equations to solve simultaneously. An alternative approach would involve substituting the pairs of values provided in the four options and working back to find the correct solution.

If students substitute the given x values they form the two equations:

$16 - 4a + b = 0$ and $4 - 2a + b = 0$. These can solved simultaneously by elimination or substitution.

Mathematics Level 4

Number and Algebra

Example: Number

What is the simplified form of the expression $\frac{x^{-1}+y^{-1}}{x^{-2}-y^{-2}}$?

A $\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}$

B $\frac{xy}{y-x}$

C $\frac{(y+x)^2(y-x)}{x^3y^3}$

D $\frac{(x-y)^2}{x+y}$

ANSWER B: $\frac{xy}{y-x}$

Students select option A: as they may rewrite the expression without negative indices but not simplify.

Students select option C: as they may make a mistake in the final stage of simplification, changing division to multiplication without inverting the second fraction.

Students select option D: as they may simplify $x^{-1}+y^{-1}$ to $(x+y)^{-1}$, and $x^{-2}-y^{-2}$ to $(x-y)^{-2}$ and then simplify.

COMMENTARY

Students are required to simplify a complex algebraic fraction with negative indices. They need to apply index laws to simplify the individual parts of the expression with negative indices and then use their knowledge of fractions to simplify the numerator and denominator. Students then need to convert the division expression into a multiplication expression and simplify.

A suggested solution process:

$$\frac{x^{-1}+y^{-1}}{x^{-2}-y^{-2}} = \frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}$$

$$= \frac{\frac{y+x}{xy}}{\frac{y^2-x^2}{x^2y^2}}$$

$$= \frac{y+x}{xy} \div \frac{y^2-x^2}{x^2y^2}$$

$$= \frac{y+x}{xy} \times \frac{x^2y^2}{y^2-x^2}$$

$$= \frac{y+x}{xy} \times \frac{x^2y^2}{(y+x)(y-x)}$$

$$= \frac{xy}{(y-x)}$$

Mathematics Level 4

Number and Algebra

Example: Algebra

The table shows costs for passengers using one of three different ride-sharing companies.

Name of company	Costs
D-Rive	$5 base fare plus $0.25 per kilometre
Mëga	$0.40 per kilometre (no base fare)
WeDrive	Fixed $10 fare for first 30 km, then an additional $0.50 per kilometre

What is the equation that describes the drive costs C (in dollars) for the distance travelled d (in kilometres), for the D-Rive company?

A $C = 0.25d + 5$

B $C = 0.4d$

C $C = 0.5d - 5$

D $C = 5d + 0.25$

ANSWER A: $C = 0.25d + 5$

Students select option B: as they may misread the question and find the equation for Mega.

Students select option C: as they may misread the question and find a partial equation for WeDrive.

Students select option D: as they may reverse the base fare (5) and cost per kilometre (0.25) for D-Rive.

COMMENTARY

Students are required to identify an equation that best describes a given word problem. Students need to identify that the costs for each company are a combination of base fare and the per kilometre rate. That is, cost (C) = base fare + per kilometre rate × distance in kilometres (d).

Students could notice that the equations that describe these costs are linear equations, where the base fare is equivalent to the y intercept and the per kilometre rate is equivalent to the gradient. Using the standard form of a linear equation $(y = mx + c)$ students could then identify the correct equation.

Mathematics Level 4

Number and Algebra

Example: Algebra

The table shows costs for passengers using one of three different ride-sharing companies.

Name of company	Costs
D-Rive	$5 base fare plus $0.25 per kilometre
Mëga	$0.40 per kilometre (no base fare)
WeDrive	Fixed $10 fare for first 30 km, then an additional $0.50 per kilometre

Over which range of distances is WeDrive the cheapest company to use?

A 0 to 20 km

B 20 to 25 km

C 25 to 40 km

D greater than 40 km

ANSWER C: 25 to 40 km

Students select option A: as they may identify that Mega is cheaper over this range; they may be confused by a transition that occurs at distance = 20km.

Students select option B: as they may identify that Mega is cheaper over this range.

Students select option D: as they may identify that D-Rive is the cheapest over this range.

COMMENTARY

Students are required to identify the range over which the WeDrive company is cheaper to use (or has smaller values). Students could use the ranges listed in the options and compare them against each of the ride-sharing companies to identify the range for which WeDrive is cheapest.

As an alternative approach, students could sketch graphs of all three functions and identify the appropriate range.

Mathematics Level 4

Statistics and Probability

Example: Statistics

The number of people present each week at a 15-week language course is shown in the stem plot.

0	4				
0	7				
1	2	4			
1	5	5	6	7	8
2	1	2	4		
2	7	7	7		

Key: 1 \| 3 = 13 people

What is most likely number of people who are enrolled in the language course?

A 7

B 18

C 27

D 266

ANSWER C: 27

Students select option A: as they may misunderstand the key and use 7 instead of 27.

Students select option B: as they may use the end value of the longest row in the stem plot.

Students select option D: as they may add the attendances for each week.

COMMENTARY

Students are required to make a reasonable interpretation of the data in a stem plot. The most likely approach will be to rule out invalid and/or unreasonable answers.

For example, A (7) and B (18) are both invalid, since there many weeks when the student numbers exceeded these values; D (266) is unreasonable because it is very large and there are no weeks when the actual number of students comes anywhere near this value.

Mathematics Level 4

Statistics and Probability

Example: Statistics

The number of people present each week at a 15-week language course is shown in the stem plot.

0	4				
0	7				
1	2	4			
1	5	5	6	7	8
2	1	2	4		
2	7	7	7		

Key: 1 \| 3 = 13 people

What is the shape of the distribution represented by the stem plot?

A Symmetrical

B Bi-modal

C Positively skewed

D Negatively skewed

ANSWER D: Negatively skewed

Students select option A: as they may assume the data is symmetrical.

Students select option B: as they may have heard of this type of shape but not understand the characteristic of shape.

Students select option C: as they may mistake the direction of the skew.

COMMENTARY

Students are required to interpret the shape of the data distribution given in the stem plot. They need to recognise that shape can be identified in stem plots by rotating the stem plot 90° anticlockwise and then interpreting the shape in the same way as they would a dot plot or histogram.

Mathematics Level 4

Statistics and Probability

Example: Statistics

The number of people present each week at a 15-week language course is shown in the stem plot.

0	4
0	7
1	2 4
1	5 5 6 7 8
2	1 2 4
2	7 7 7

Key: 1 \| 3 = 13 people

What is the mean number of people who are present at the course each week?

Write your answer to 1 decimal place.

[ ] people

ANSWER 17.7 people

COMMENTARY

Students are required to determine the mean of the data set by adding all of the individual values and then dividing by the total number of data values.

The mean is given by:

$$\frac{4+7+12+14+15+16+17+18+21+22+24+27+27+27}{15} = \frac{266}{15} = 17.7333$$

The answer to 1 decimal place is 17.7

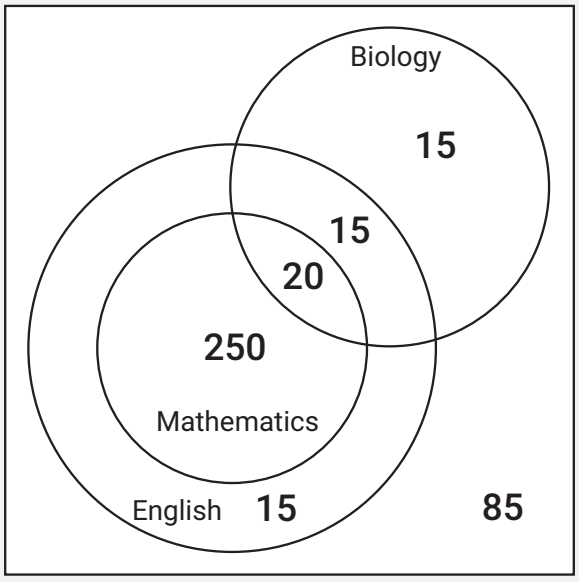
Mathematics Level 4

Statistics and Probability

Example: Probability

A senior secondary college has 400 students.

The number of students studying Biology, English and Mathematics is shown in the Venn diagram



What is the probability that a randomly chosen student studies Biology **and** English but **not** Mathematics?

A 0.0375

B 0.05

C 0.0875

D 0.1125

ANSWER A: 0.0375

Students select option B: as they may find the probability that a student studies all three subjects (20/400).

Students select option C: as they may combine the students that study Biology/English with those who study Biology/English/Mathematics (35/400).

Students select option D: as they may combine all Biology and English students (45/400).

COMMENTARY

Students are required to interpret the Venn diagram to calculate the probability of a student studying Biology and English but not Mathematics. They need to recognise that the target group represents the intersection between the Biology and English sections of the diagram (15).

The probability is $\frac{15}{400} = 0.0375$

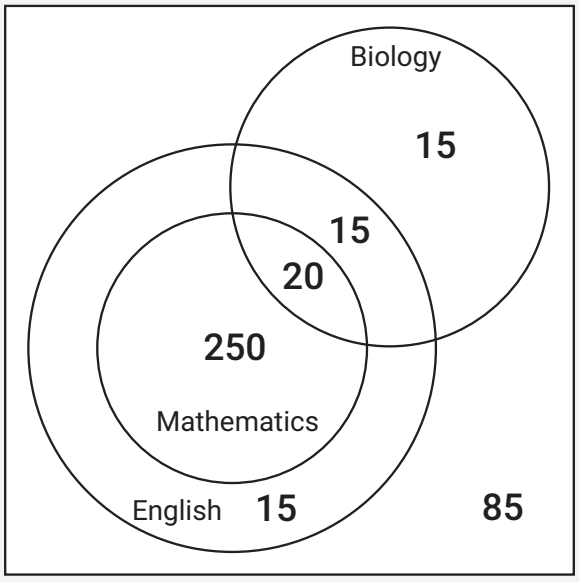
Mathematics Level 4

Statistics and Probability

Example: Probability

A senior secondary college has 400 students.

The number of students studying Biology, English and Mathematics is shown in the Venn diagram



What is the probability that a randomly chosen student who studies Biology **also** studies Mathematics?

A 0.05

B 0.125

C 0.4

D 0.7

ANSWER C: 0.4

Students select option A: as they may find the probability of a student studying Biology and Mathematics out of the total number of students (20/400).

Students select option B: as they may find the probability of a student studying Biology out of the total number of students (50/400).

Students select option D: as they may find the probability of a Biology student also studying English and Mathematics (35/50).

COMMENTARY

Students are required to interpret the Venn diagram to solve a conditional probability problem. They need to identify that the number of students studying Mathematics are to be selected from a smaller sample – those who also study Biology.

The probability is $\frac{20}{50} = 0.4$

ACER Academy

Annotated test questions

Reading Level 4

Reflecting on the Text

Reflect on form

The following passage is taken from a longer essay about human and animal intelligence.

What truly separates the human animal from other, lesser species? The answer seems so self-evident as to not warrant serious consideration. Indeed the very phrase 'human animal' strikes many as a contradiction it terms: 'human' is precisely that which is not animal. Nevertheless, let us take the question seriously – if only for a moment – and consider which uniquely human trait it is that so elevates us above all other living things.

Our intelligence is surely the first consideration one gives. We can point to a wealth and history of human invention: architecture, science, industry, art, mathematics and so on. What animal, we ask, could build the Duomo in Florence, or deduce the sum of angles in a triangle? What moose could write librettos for the opera? Which guinea pig could split the atom? It must surely be our capacity for intelligence that sets us humans apart.

While this is indeed true, what are those unique traits that so contribute to our supreme intelligence? Perhaps our considerable brain size provides a clue. The human brain is much larger than those of many other species, hence our intelligence, yet a giraffe's brain is about the same size, and whales' and elephants' brains are enormous compared to ours. Even if we consider brain size relative to body size, mice and shrews have bigger brains than we do. Are they more intelligent than us?

What about tool use? Is it that we as humans have been able to craft useful implements like axes and spears (and now robots and computers) with which to manipulate and master the natural world to our advantage? No other species can lay claim to such technological ingenuity as we can. Even so, the trait of tool use in-and-of-itself is not unique to humans. Chimpanzees are known to use giant leaves as umbrellas and create spears with which to hunt. Elephants create fly-swats with branches. Even otters use rocks as hammers to open shellfish to eat.

The writer's use of questions in paragraph 2

A reveals his genuine disregard for the intelligence of animals.

B reveals his own lack of interest in the question of animal intelligence.

C caricatures a common understanding of the superiority of human intelligence.

D suggests he wonders whether animals might be as intelligent as humans.

Answer: C. caricatures a common understanding of the superiority of human intelligence.

Explanation

The repetitive use of rhetorical questions in the second paragraph is a device that generates a condescending and dismissive tone about animal intelligence when compared to humans. Further reading of the passage however, reveals this is unlikely to reflect the author's actual point-of-view. The fact that the author seeks – in the rest of the passage – to explore the differences between animal and human intelligence, and highlights similarities, would suggest he is using irony in paragraph 2, to caricature as overly simple a commonly-held understanding humans have their own intelligence. The author setting up a misconception at the outset that he later exposes through further discussion is a common device used in essays of this kind.

Text complexity

Though the text does not use technical vocabulary or jargon, there is a relatively high use of nominalised words and abstract nouns: 'consideration', 'contradiction', 'capacity' and 'ingenuity'; less frequently-used verbs – 'warrant', 'elevates' or 'manipulate' for example – and a more formal register overall, adding to the complexity of the passage. The structural features of the text, such as the writer appearing to engage himself in a dialogue through constantly questioning and answering himself, may confuse a reader who is unfamiliar with this trope of writing. This is further complicated by the dismissiveness of the tone at the outset, which does not appear to reflect the author's own view when one reads on, but is instead an instance of irony.

Reading Level 4

Reflecting on the Text

Reflect on ideas

The following passage comes from the prologue to a textbook on calculus.

Calculus Made Easy

What one fool can do, another can.
(Ancient Simian Proverb.)

PROLOGUE.

Considering how many fools can calculate, it is surprising that it should be thought either a difficult or a tedious task for any other fool to learn how to master the same tricks.

Some calculus-tricks are quite easy. Some are enormously difficult. The fools who write the textbooks of advanced mathematics – and they are mostly clever fools – seldom take the trouble to show you how easy the easy calculations are. On the contrary, they seem to desire to impress you with their tremendous cleverness by going about it in the most difficult way.

Being myself a remarkably stupid fellow, I have had to unteach myself the difficulties, and now beg to present to my fellow fools the parts that are not hard. Master these thoroughly, and the rest will follow. What one fool can do, another can.

The author frequently makes references to 'fools' in this passage in order to

A suggest that mathematicians lack intelligence.

B suggest that calculus is not as difficult as it appears.

C dissuade the reader from attempting to understand calculus.

D persuade the reader that studying calculus is a waste of one's time.

Answer: B. suggest that calculus is not as difficult as it appears.

Explanation

In order to correctly identify the function of a single element within the passage, the student is required to consider the purpose of the whole text – to clearly explain calculus to the reader. This requires the reader to suspend the idea that the author is using the word 'fool' for its usual, pejorative purpose, as it performs an unusually positive function within the text: to encourage those who might find calculus difficult. Therefore (B) is correct. Options (C) and (D) miss the overall intention of a textbook that aims to make calculus 'easy', whilst option (A) misses the irony of the word's use when applied to mathematicians.

Text complexity

This text is mostly composed of familiar words, though with some reasonably low-frequency words such as 'tedious' and 'seldom', and some relatively antiquated and stylistically unusual phrasing, such as 'desire to impress you', 'Being myself' and 'now beg to present to' that suggest the age of the text. Students may be familiar with the formal function of a prologue, but may have less familiarity with texts that employ epigraphs. Despite a relative lack of complexity of ideas or information for this level, the text is subtly ironic in employing seemingly paradoxical ideas of 'clever fools', and is even quite self-deprecating in this irony.

Reading Level 4

Reflecting on the Text

Reflect on ideas

The following passage comes from Heart of Darkness (1899) a novel by Joseph Conrad, set in a jungle in central Africa in the late 19th century. The narrator – an English steamboat captain – describes his experience of travelling upriver in this jungle.

Going up that river was like traveling back to the earliest beginnings of the world, when vegetation rioted on the earth and the big trees were kings. An empty stream, a great silence, an impenetrable forest. The air was warm, thick, heavy, sluggish. There was no joy in the brilliance of sunshine. The long stretches of the waterway ran on, deserted, into the gloom of overshadowed distances. On silvery sand-banks hippos and alligators sunned themselves side by side. The broadening waters flowed through a mob of wooded islands; you lost your way on that river as you would in a desert, and butted all day long against shoals, trying to find the channel, till you thought yourself bewitched and cut off for ever from everything you had known once—somewhere—far away—in another existence perhaps. There were moments when one's past came back to one, as it will sometimes when you have not a moment to spare for yourself; but it came in the shape of an unrestful and noisy dream, remembered with wonder amongst the overwhelming realities of this strange world of plants, and water, and silence. And this stillness of life did not in the least resemble a peace.

The passage mostly suggests that the narrator found the river expedition to be

A tedious and boring.

B spectacular and magical.

C unsettling and menacing.

D exciting and adventurous.

Answer: C. unsettling and menacing.

Explanation

This question requires the reader to link various details about the narrator's experiences of the jungle landscape and his travels through it, in order to form an accurate impression of how he experiences it emotionally. The connotations of individual details in the passage are sometimes not immediately apparent and are open to some interpretation; at various, isolated moments in the text, the narrator's observations could plausibly be interpreted as suggesting his excitement, boredom, wonderment or even peaceful contentment. However when taken together, along with the negatively connoted references to 'unrestful' dreams, travelling 'into the gloom', there being 'no joy' in the sunshine, being 'bewitched' and 'cut off' from his previous existence and repeated references to stillness, silence and emptiness, the reader can infer the narrator feels unnerved, and thus (C) is shown to be the correct answer.

Text complexity

The vocabulary demands of the passage are relatively low for this level. Words such as 'shoal' and 'channel' have specific scientific meanings in reference to the river that may be unfamiliar, and a few low-frequency words such as 'impenetrable' and 'bewitched' do appear. However, the length and number of clauses in some of the important later sentences in the passage make them quite dense, placing more cognitive demands on the reader as they try to track meaning. The register of the text is quite formal, as shown in some turns of phrase such as 'one's past came back to one'. Further, some ambiguity exists at the outset of the passage; the imagery within the opening sentence – whilst clear enough – is not initially discernible as positively or negatively connoted. It only resolves as negatively-connoted imagery (signifying dangerous, untamed wilderness) as one reads further into the passage.

ACER Academy

Annotated test questions

Reading Level 4

Interpreting Meaning

Analyse multiple perspectives

The Example

Here's an example from
A Butterfly;
That on a rough, hard rock
Happy can lie;
Friendless and all alone
On this unsweetened stone.

Now let my bed be hard
No care take I;
I'll make my joy like this
Small Butterfly;
Whose happy heart has power
To make a stone a flower.

W.H. Davies

Which of the following sayings is closest in meaning to the poem?

A "Beauty is in the eye of the beholder."

B "Of all possessions a friend is the most precious."

C "The greatest wealth is to live content with little."

D "A rose by any other name would smell as sweet."

Answer: C. "The greatest wealth is to live content with little."

Explanation

The reader is first required to understand the poem's meaning as it is developed through metaphor. The reader then needs to identify a sophisticated paraphrase of the poem's meaning in one of several clichéd sayings. Each of the sayings have some plausible thematic link to, or shared reference in, the poem. In order to identify the correct option, readers need to resist making simple word-matches or obvious symbolic links between the poem and the sayings, and attend more to the poem's overall meaning, (what 'example' the butterfly is providing). This reveals (C) to be the correct answer. A reader with a limited understanding of the meaning of the poem and the sayings might make a more simplistic, superficial connection based on similar imagery or word matches.

Text complexity

The phrasing and syntax of some lines in the poem ('Happy can lie', 'No care take I') may require some more deliberate decoding. The poet also employs figurative language such as metaphor and a degree of anthropomorphism rather than direct, literal statements of meaning.

Though each option is quite short and composed entirely of common, high-frequency words, the task of comparing the ideas across and between the five texts increases the textual complexity of the task itself.